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Investigating Covariational Reasoning: What Do Students Show when Solving Mathematical Problems?

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Investigating Covariational Reasoning: What Do Students Show when Solving Mathematical Problems?

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Abstract. Covariational reasoning plays a significant role for solving problems. This study examines the covariational reasoning of master program students when solving mathematical problems such as fraction, velocity and acceleration, proportion, and integral. 26 students of mathematics education master program, Universitas Negeri Surabaya, are involved in this study. Generally, mental action of students is more prominent on fraction and proportion than on the other issues. On fraction and proportion problems, most students are able to fulfill all mental actions such as coordinating the value, the direction, and the amount of change of one variable, and also coordinating the average and the instantaneous rate of change of the function. However, on velocity, acceleration, and integral problems most students cannot show their mental actions well. They only fulfill 3 of 5 mental actions of covariational reasoning. Generally, the shape of their graph related to those problems are irrespective with initial point. These findings suggest that learning in mathematics should place increased emphasis on problem involving graph to promote covariational reasoning of the students.

1. Introduction

It has been recognized that one of primary aims of education is to foster student's reasoning which isthe main activity in mathematical activities [1]. Therefore, it becomes crucial task for mathematics teachers to involve reasoning activities in mathematical learning and make mathematics meaningful. Thus, the ability to reason which is an important achievement for mathematics education students, especially prospective mathematics teachers can be fulfilled.

One type of reasoning that is closely related to mathematical problem solving is covariational reasoning. It is defined as cognitive activity which involves coordinating two varying quantities while attending to the ways in which they change in relation to each other [2, 3]. Covariational reasoning provides powerful insight and builds powerful mechanisms to improve reasoning abilities in solving mathematical problems [4].

Based on study result of [3], it found 5 levels of covariational reasoning development. A level will be reached only if underneath level has been fulfilled. For each level, there is indicator as follows:

Table 1. Level and Indicator of Covariational Reasoning

Level	Indicators
Coordination	Coordinating the value of one variable with changes in the other

Direction	Coordinating the direction of change of one variable with changes in the other variable
Quantitive Coordination	Coordinating the amount of changes of one variable with changes in the other
Average Rate	Coordinating the average rate of change of the function with uniform increments of change in the input variable
Instantaneous Rate	Coordinating the instantaneous rate of change of the function with continuous changes in the independent variable for the entire domain of the function

Some previous studies have shown relationship between covariational reasoning and calculus [5, 6, 7, 8]. According to [3], covariational reasoning is foundational for understanding major concepts of calculus. This is because the main of the material studied in calculus is related to the activity of analyzing the speed of a change. The activity is closely related to the activity of coordinating changes in quantity which is a covariational reasoning activity. This study examines student's covariational reasoning in solving calculus problems such as velocity and acceleration, integral, fraction, and proportion.

Moreover, research on covariational reasoning becomes crucial to study, reference to [9] who investigated the covariational reasoning abilities of a group of mathematics teachers and emphasizes that even though covariational reasoning plays an important role but in fact, not only teachers' covariational reasoning abilities were weak and lack depth but also their predictions about students' reasoning abilities bounded by their own thoughts related to the problem. Research on covariational reasoning is expected to provide important input to build covariational reasoning capabilities for both teachers and students.

2. Method

A Qualitative approach was used in this study to investigate student's covariational reasoning in solving mathematical problems which formed velocity and acceleration, integral, fraction, and proportion questions. A purposive sampling technique was applied to select the subjects (26 postgraduate students of mathematics education from Universitas Negeri Surabaya). All of students were given covariational reasoning test in order to promote as much student's covariational reasoning as possible, then were individually interviewed based on their answer. The obtained data was qualitatively analyzed based on covariational reasoning indicator which was done through three steps including data condensation, data presentation, and data interpretation.

3. Result and Discussion

The covariational reasoning tests are described below.

Velocity and Acceleration
1. Look at the picture of Andre's route from City A to City B below.
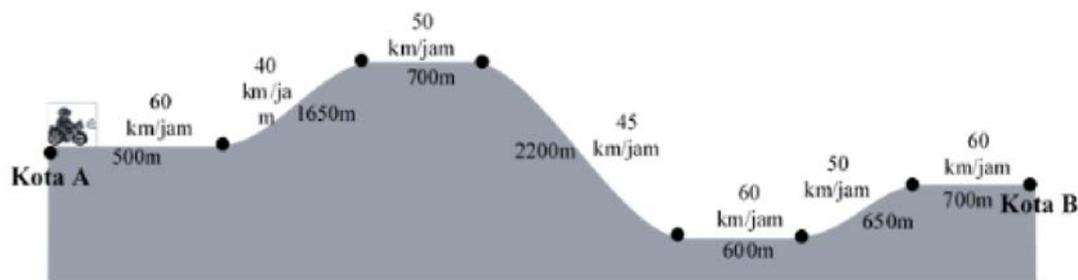
Describe in your words, the speed change of the motorbike driven by Andre from city A to city B! If the acceleration of the motor is expressed by the change of motor speed per unit of time, what is the average acceleration of the motor that Andre drives from city A to city B? How much time does Andre need from city A to reach city B?

2. Look at the picture of Maya's route from City X to City Y below.
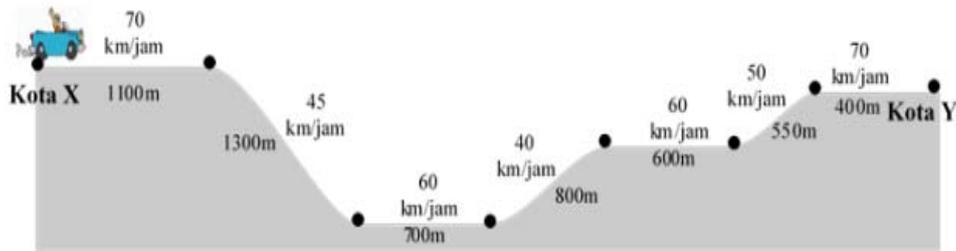
Describe the speed change of the car driven by Maya from city X to city Y! If the acceleration of the car is expressed by the change of car speed per unit of time, what is the average acceleration of the car that Maya drives from city X to city Y? How much time does Maya need from city X to reach city Y?

Fraction
1. Mom divides a tart into 5 equal portions. Dito gets one part, while Dera gets 2 parts.
    a. In fraction form, how many parts of cake do Dito and Dera get?
    b. Then, Dito and Dera divide their tart into 2 equal parts. Each of them give 1 part of their cake to Dini. In fraction form, how many parts of cake does Dini get?
    After being given to Dini, Who gets more cake? (Dito, Dera or Dini)? Please explain it!
2. Amir and Badrul have the same amount of marbles. Each has red and blue marbles. 2/3 of all Badrul marbles are red and the rest are blue. While 2/5 parts of the number of Badrul's red marbles are the same as the number of Amir's red marbles. Apparently, each of them lost one red marble.
    a. Whose red marbles are more after being lost?
    b. How many parts red marbles of Amir and Badrul after each of their marbles disappear?
3. A grocery store holds a promo for purchasing eggs. Each egg buyer will get a bonus as much as 1/5 kg from the store. Ani buys 5/4 kg of eggs and Dian buys 3/2 kg of eggs. The price of eggs is IDR 24,000/kg.
    a. How many kg of eggs did Ani and Dian buy before getting a bonus?
    b. Who gets more eggs after getting a bonus? Please try to explain it!
   c. How much difference of money must Dian and Ani pay?

Proportion
1. Out of 100 students, half of them keep dogs and the rest of them keep cats. Based on your opinion, which animals are the most kept by 100 students? If 2/5 students who keep dogs are male, how many female students keep cats? If the number of male students who maintain dogs are equals to the number of female students who maintain dogs, what is the ratio of the number of male students to the number of female students who have pets?
2. In a stall, Agus buys $\frac{7}{5}$ kg sugar, while Berta buys $\frac{5}{3}$ kg sugar. Each of them plans to give $\frac{1}{2}$ kg sugar to their mom. The price of 1 kg of sugar is Rp 15.000,-.
    a. How many kg of sugar did Agus and Berta buy before each of their sugar was given to his mother?
    b. After giving it to her mother, who has more sugar?
    c. How much money do Agus and Berta have to pay to buy the sugar?

Integral
1. A swimming pool is initially contained of 900 m^3 water. The pool is connected to 2 types of pipes, namely pipes to fill water into the pool and pipes to remove water from the pool. $Q(t)$ states the change of water flow at time t.
    $$Q(t) = \{t\,3\ -3t + 15\ -t + 5\,t - 7 \quad if\ 0 \le t \le 3\ if\ 3 < t \le 4\ if\ 4 < t \le 6\ if\ 6 < t \le 7\ if\ 7 < t \le 10$$

If $Q(t) = \frac{dV(t)}{dt}$, sketch graph $V(t)$ which states the volume of water at time t in the pool for 10 hours!

2. An object is at its starting point ($t = 0$). Graph $v(t)$ expresses the movement speed of an object (in $\frac{meter}{second}$).
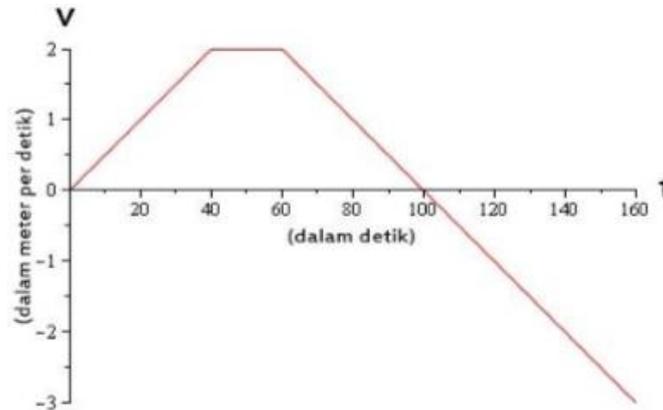
If $v(t) = \frac{ds(t)}{dt}$, sketch graph $s(t)$ which states the distance traveled by object from $t = 0$ to $t = 160$!

3. The swimming pool is initially contained 400 m^3 water. The pool is connected to 2 types of pipes, namely pipes to fill water into the pool and pipes to remove water from the pool. The following table is the result of observing changes in water flow for 7 hours. The observation begins at 10:00 and ends at 17:00.

Observation time	Time (t) in hour	Changes in water flow (counted from initial volume $t = 0$) $X(t)$ in m^3 per hour
10.00	0	0
11.00	1	1
12.00	2	2
13.00	3	3
14.00	4	3
15.00	5	0
16.00	6	-1
17.00	7	0

If $X(t) = \frac{dY(t)}{dt}$, sketch graph $Y(t)$ which expresses volume of water at time t in the pool for 7 hours!

A. *Student's Covariational Reasoning on Velocity and Acceleration Problem*

The covariational reasoning of students were explored to determine how students' fulfillment the level of covariational reasoning in solving velocity and acceleration problem. Figure 1 shows the percentage of students who fulfilled the covariational reasoning level. 8% of students were able to have all the covariational reasoning.
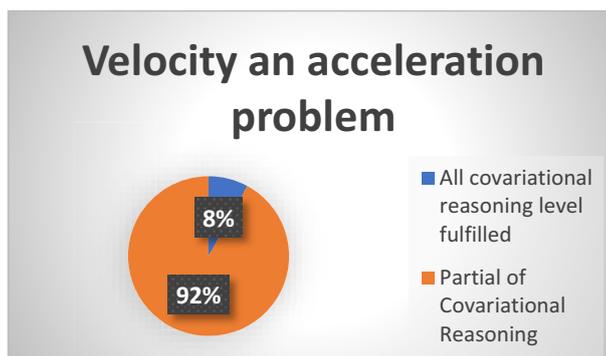
Figure 1. Covariational reasoning level on velocity and acceleration problem

On coordination level, to coordinate the output values (average acceleration and overall travel time) as the values change in input (speed and distance between two points), all students divide the presented data into several intervals. The first interval marks the input that is valid at the first point/beginning of departure until the second point. This interval is used as the basis for determining the first output. The second interval marks the input that applies at the second point to the third point and it is used as the basis for determining the second output, and so on. In this case, input acts as the independent variable while output acts as the dependent variable. This is analogous to the findings of [3] that indicate mental action on coordination level.

On direction level, to coordinate the direction of changes in output as changes in input, 1-2 students identify the presence of output (acceleration) when changes in speed and time changes are both positive or negative, and output (deceleration) appears when changes in speed or changes in time are negative, while other students did not identify this condition. However, in this case all students identify the direction of output (speed/deceleration of travel time) depending on distance and speed. The travel time will be faster if the travel speed is greater than the mileage and slower if the mileage is greater than the travel speed.

On quantitive coordination level, to coordinate the magnitude of changes in output with changes in input, only 1-2 students determine the magnitude of the acceleration/deceleration value by dividing the value of the speed change by the value of the time change for each interval. Nevertheless, all students divide the value of travel distance by the value of speed travel to determine the amount of travel time. To determine the total travel time, students add up the travel time for all intervals. On average rate level, to coordinate the average change in output as changes in input, all students add up the value of the acceleration/deceleration of all intervals and divide by the number of intervals to get the average acceleration/deceleration. For the last level, instantaneous rate, to coordinate instant changes in output (acceleration/deceleration and travel time) when input changes continuously, all students take a look at the output value at the end of each predetermined interval. In this case, all students review temporary travel time. But, only 1-2 students who also review the values of acceleration/deceleration.

B.   Student's Covariational Reasoning on Fraction Problem

Fraction problem is considered as more friendly problem than velocity and acceleration problem for students. Figure 2 shows the fulfillment of students in solving fraction problem.
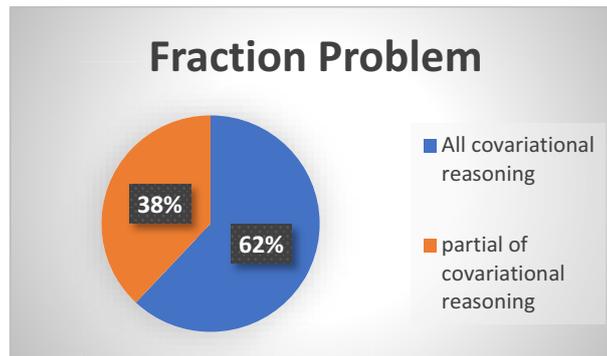
Figure 2. Covariational reasoning on Fraction Problem

On fraction problem, more than 90% students answer the question. Differences in student answers can be categorized into high group and under low group. On coordination level, to coordinate the output values as the values change in input, all students present their parts in fraction form, represent it to images, and divide the condition to be some parts. On direction level, to coordinate the direction of changes in output as changes in input, high categorized students identify that the output will increase when there is addition and will decrease when there is reduction or division. This is analogous to the findings of [3] that "Constructing an increasing straight line" to mark mental actions 2. Meanwhile, under low categorized students did not do it.

On quantitive coordination level, to coordinate the magnitude of changes in output with changes in input, high categorized studens add, subtract, or divide all inputs according to the instructions in the problem. But, under low students didn't do it because of lack of understanding on questions. On average rate level, to coordinate the average rate of change in output as changes in input, more than half of the students determine the rate of output quantity according to the sum value, subtraction value, or divider value. For the instantaneous rate level, to coordinate instant changes in output when input changes continuously, all students take a look at the output value at the end of each predetermined condition.

C. Student's Covariational Reasoning on Proportion Problem

More than 88% of students show their solution on proportion problem. Figure 3 shows the fulfilment of students' covariational reasoning on proportion problem.
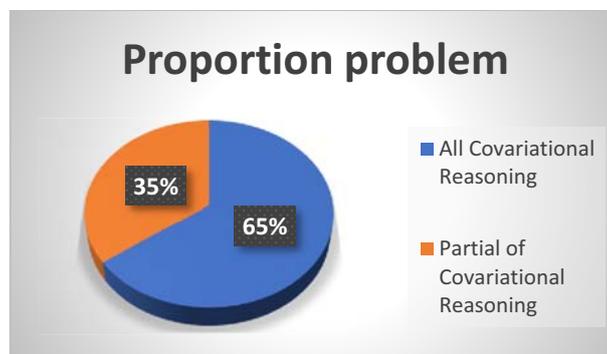

Figure 3. Covariational Reasoning on Proportion Problem

Student's answer can be categorized into 2 groups, namely high and under low. Almost all students can demonstrate the application of the proportion concept well. On coordination level, to coordinate the output values as the values change in input, all students divide the condition to be some parts. On direction level, to coordinate the direction of changes in output as changes in input, high categorized students identify that the output will increase when there is addition and will decrease when there is reduction. They also determine the value of the output in each condition given to the problem, either by adding up or subtracting inputs with the stated values. This condition shows quantitive coordination level. This is analogous to the findings of [3] "Plotting points / constructing secant lines" marks mental actions 3. For the last two level, only a few students who are under low categorized did not show those mentioned behavior.

On average rate level, to coordinate the average rate of change in output as changes in input, more than 80% of the students determine the rate of output quantity according to the condition given such as proportion value. This condition is analogous to the findings of [3], "Constructing contiguous secant lines for the domain" marks mental action 4. For the last level, to coordinate instant changes in output when input changes continuously, students determine output at the end of each predetermined condition.

D. Student's Covariational Reasoning on Integral Problem

Integral problem is considered as the most challenging problem for students. Roundly, just around 15% students fulfill some indicators of covariational reasoning.

Table 2. The Covariational reasoning on Integral Problem

Coordination Level	Direction Level	Quantitative Coordination
Draw axis lines on Cartesian coordinates	Construct the up and down function	Substitute the value of t on the function

To be more specific, on coordination level, to coordinate the output values as the values change in input, students make axis lines on cartesian coordinates. Horizontal lines to mark input values while vertical lines to mark output values. Input acts as an independent variable, while output acts as a dependent variable. On direction level, to coordinate the direction of changes in output as changes in input, students construct the up and down functions (Q(t), V(t), and X(t)) but not construct the asked function. On quantitive coordination level, to coordinate the magnitude of changes in output with changes in input, students substitute the value of t on Q(t), V(t), and X(t) then plot the dots on the cartesian coordinates.

On average rate level, to coordinate the average change in output as changes in input, students connect the points that indicate the value of the function Q(t), V(t), and X(t) which are not the final functions in question. For the last level, instantaneous rate, to coordinate instant changes in output when input changes continuously, students construct a smooth curve that shows the value of Q(t), V(t), and X(t) which are not the final functions in question. The last two conditions complement and appear to support previous findings by researchers such as [3, 10] who find that at the college level, majority of students have difficulty in showing covariational reasoning behavior that is higher than direction level (level 2) or quantitive coordination level (level 3).

4. Conclusion

Overall, students show their covariational reasoning performance well on fraction and proportion problems. For both problems, more than 85% students answer the questions given. The opposite condition appears precisely on two other problems, especially integral. Students only show 3 out of 5 mental actions that correspond to indicators of covariational reasoning. At the third level, students only determine one of two outputs that should be determined. The output determined by the student is the initial output that will

be used to determine the final output value. At the fourth and final level, students only focus on the initial output. While the final output becomes the accent at these levels.

References

[1]   Ferrini-Mundy J 2000 Principles and standards for school mathematics: A guide for mathematicians *Notices of the American Mathematical Society* **47**(8)

[2]   Saldanha L and Thompson P W 1998 Re-thinking covariation from a quantitive perspective: Simultaneous continuous variation, in *Proceedings of the Annual Meeting of the Psychology of Mathematics Education-North America. Raleigh, NC*

[3]   Carlson M, Jacobs S, Coe E, Larsen S, and Hsu E 2002 Applying covariational reasoning while modeling dynamic events: A framework and a study *Journal for Research in Mathematics Education* pp 352-378

[4]   Strom A D 2006 The Role of Covariational Reasoning in Learning and Understanding Exponential Functions, in *Proceedings of the Twenty-Eighth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* **2** pp 624-630

[5]   Thompson P W 1994 Images of rate and operational understanding of the fundamental theorem of calculus *Educational studies in mathematics* **26**(2-3) pp 229-274

[6]   Cottrill J, Dubinsky E, Nichols D, Schwingendorf K, Thomas K, and Vidakovic D 1996 Understanding the limit concept: Beginning with a coordinated process scheme *Journal of mathematical behavior* **15**(2) pp 167-192

[7]   Zieffler A S and Garfield J B 2009 Modeling the growth of students'covariational reasoning during an introductory statistics course *Statistics Education Research Journal* **8**(1)

[8]   Moore K C, Paoletti T, and Musgrave S 2013 Covariational reasoning and invariance among coordinate systems *The Journal of Mathematical Behavior* **32**(3) pp 461-473

[9]   Zeytun A S, Cetinkaya B, and Erbas A K 2010 Mathematics Teachers' Covariational Reasoning Levels and Predictions about Students' Covariational Reasoning Abilities. *Educational Sciences: Theory and Practice* **10**(3) pp 1601-1612

[10]   Koklu O 2007 *An Investigation of College Students' Covariational Reasonings* (Doctoral dissertation, Florida State University).